Multiple-Frequency Operation of Y-Junction Circulators Using Single-Mode Approximation

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Abstract—**The conditions for a Y-junction circulator to operate within more than one frequency band are studied. By taking into account only the dominant resonance mode at each circulation frequency, explicit expressions are obtained which define the properties of the circulator. The results are presented in diagrams and tables which enable rapid evaluation of obtainable performance for some of the most important mode combinations. The validity of the results are experimentally verified.**

I. Introduction

It is a well-known fact that higher order mode operation of Y-junction circulators is possible if the proper adjustments to the circulation conditions are made [1]. The possibility of obtaining circulation at two frequencies using different dominant modes was also discovered early [2], [3]. Recently, the double circulation frequency operation has been discussed by Nagao [4]. By using Y junctions loaded with dielectric-ferrite composites, he demonstrates the possibility to achieve broadband circulation with the modes 1 and 1A according to Davies and Cohen [5].

In this paper the approximation of taking into account only the dominant mode is applied. Thereby, it has been possible to find an analytic solution to the equations which for given mode combinations describe the circulation conditions at the different center frequencies. The results are expressions defining, for instance, the saturation magnetization of the material and the bias field to be used when the circulation frequencies are given. A new relation which gives the loaded Q values of the junction for different modes is obtained. Explicit expressions are presented which define the properties of the circulator and conditions which have to be satisfied for an arbitrary number of circulation frequencies.

II. Theory

A. Conditions for Circulation and Relations for the Input Impedance

The impedance matrix of a loss-free rotationally symmetrical three-port junction can be written

$$[Z]=\begin{bmatrix} Z_{11} & Z_{12} & -Z_{12}^* \\ -Z_{12}^* & Z_{11} & Z_{12} \\ Z_{12} & -Z_{12}^* & Z_{11} \end{bmatrix}. \tag{1}$$

In the case of a junction circulator, the elements of the matrix can be expressed as [6]

$$Z_{11}=\sum_{n=-\infty}^{\infty} Z_n \tag{2}$$

$$Z_{12}=\sum_{n=-\infty}^{\infty} Z_n e^{j2n\pi/3} \tag{3}$$

where

$$Z_n=j\frac{Z_e \sin^2(n\Psi)J_n(kR)}{Z_d\pi n^2\Psi[J_n'(kR)-(\kappa n/\mu kR)J_n(kR)]} \tag{4}$$

and

$J_n(kR)=$ Bessel function of the first kind

$k=\omega\sqrt{\epsilon_0\epsilon_f\mu_0\mu_e}$ = radial wave propagation constant

$R=$ radius of the junction

$\Psi=$ half the coupling angle

$Z_e=\sqrt{\mu_0\mu_e/\epsilon_0\epsilon_f}$ = intrinsic wave impedance of the ferrite

$Z_d=\sqrt{\mu_0\mu_d/\epsilon_0\epsilon_d}$ = intrinsic wave impedance of the surrounding material

$\epsilon_f=$ specific permittivity of the ferrite

$\epsilon_d=$ specific permittivity of the surrounding material

$\mu_e=(\mu^2-\kappa^2)/\mu=$ specific permeability of the ferrite

$\mu_d=$ specific permeability of the surrounding material

$\mu,\kappa=$ elements of the Polder tensor of the ferrite.

For small magnetic splitting, it is often justified to make the approximation of taking into account only the dominant mode pair for describing the properties of the junction close to resonance [6]. The approximate expressions for Z_{11} and Z_{12} are under these conditions

$$\begin{cases} Z_{11}=Z_n+Z_{-n} & (5)\\ Z_{12}=Z_n e^{j2n\pi/3}+Z_{-n}e^{-j2n\pi/3} & (6)\end{cases}$$

or expressed in the original variables

$$\begin{cases} Z_{11}=jZ_{n1}J_n'(kR) & (7)\\ Z_{12}=Z_{n1}\left[-\frac{\kappa n}{\mu kR}J_n(kR)\sin(2n\pi/3)+jJ_n'(kR)\cos(2n\pi/3)\right] & (8)\end{cases}$$

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where

$$Z_{n1}=\frac{2Z_e\sin^2(n\Psi)J_n(kR)}{\pi Z_d n^2\Psi\left\{J_n'(kR)^2-\left[\frac{\kappa n}{\mu kR}J_n(kR)\right]^2\right\}}. \quad (9)$$

The conditions for circulation are found from (1), assuming the third-port isolated

$$Z_{11}+\frac{Z_{12}^2}{Z_{12}^*}=1 \quad (10)$$

which with (7) and (8) gives

$$\begin{cases} kR=X & (11)\\ \dfrac{2Z_e\sin^2(n\Psi)\mu}{\pi Z_d n^3\Psi\kappa}X\sin(2n\pi/3)=1, & (12)\\ n\neq 0,3,6,\cdots \end{cases}$$

where X is the mth solution of $J_n'(X)=0$.

With the opposite circulation direction, we get, correspondingly, the left-hand side of (12) equal to minus one. Thus the circulation direction is given by the sign of [sin $(2n\pi/3)\cdot\mu/\kappa$].

The exact expression for the normalized input impedance Z_{in}, using (1), is [1]

$$Z_{in}=Z_{11}+\frac{Z_{12}^3-Z_{12}^{*3}+2Z_{12}Z_{12}^*(Z_{11}+1)}{(Z_{11}+1)^2+Z_{12}Z_{12}^*}. \quad (13)$$

Inserting Z_{11} and Z_{12} according to (7) and (8) into (13), and observing that near the circulation points $|\mathrm{Re}\{Z_{12}\}|$ is close to 1 while Z_{11} and $\mathrm{Im}\{Z_{12}\}$ are close to zero, we get

$$Z_{in}\approx Z_{12} \quad (14)$$

or

$$Z_{in}\approx -Z_{12}^* \quad (15)$$

depending on the circulation direction. These relations are exact at circulation and good approximations in a close vicinity of circulation. They will be used to derive the loaded Q value of the junction related to different resonance modes.

B. Design for Double-Frequency Circulation

The conditions for circulation at the frequencies f_1 and f_2 are, by using (11) and (12),

$$2\pi f_1\sqrt{\epsilon_0\epsilon_f\mu_0\mu_{e1}}\,R=X_1 \quad (16)$$

$$2\pi f_2\sqrt{\epsilon_0\epsilon_f\mu_0\mu_{e2}}\,R=X_2 \quad (17)$$

$$\frac{2Z_{e1}\sin^2(n_1\Psi)\mu_1}{\pi Z_{d1}n_1^3\Psi\kappa_1}X_1\sin(2n_1\pi/3)=S_1 \quad (18)$$

$$\frac{2Z_{e2}\sin^2(n_2\Psi)\mu_2}{\pi Z_{d2}n_2^3\Psi\kappa_2}X_2\sin(2n_2\pi/3)=S_2 \quad (19)$$

where the indexes 1 and 2 are related to the first and the second circulation, respectively, and $S(=1$ or $-1)$ defines the circulation direction. If we divide (17) by (16), and (19) by (18), we get

$$\frac{f_2}{f_1}=\frac{X_2}{X_1}\sqrt{\frac{\mu_{e1}}{\mu_{e2}}} \quad (20)$$

and

$$\frac{X_2 n_1^3\sin^2(n_2\Psi)\mu_2\kappa_1\sin(2n_2\pi/3)}{X_1 n_2^3\sin^2(n_1\Psi)\mu_1\kappa_2\sin(2n_1\pi/3)}\sqrt{\frac{\mu_{e2}\mu_{d1}}{\mu_{e1}\mu_{d2}}}=S_{1,2} \quad (21)$$

where $S_{1,2}=S_2/S_1$. For small Ψ, (21) can be reduced to

$$\frac{X_2 n_1\mu_2\kappa_1\sin(2n_2\pi/3)}{X_1 n_2\mu_1\kappa_2\sin(2n_1\pi/3)}\sqrt{\frac{\mu_{e2}\mu_{d1}}{\mu_{e1}\mu_{d2}}}\approx S_{1,2}. \quad (22)$$

It can be noticed that $S_{1,2}=1$ means equal circulation directions while $S_{1,2}=-1$ means opposite ones.

We will now introduce the frequency dependence of the variables in (20) and (21). Generally, it is possible to use expressions for the tensor permeability parameters which include the behavior in the partially magnetized state [7], [8]. For the purpose of simplicity, however, we will here treat only the case when the material is magnetically saturated. In the lossless case, the elements μ and κ of the permeability tensor are then given by [9]

$$\mu_{1,2}=1+\frac{f_m f_0}{f_0^2-f_{1,2}^2} \quad (23)$$

$$\kappa_{1,2}=\frac{f_m f_{1,2}}{f_0^2-f_{1,2}^2} \quad (24)$$

where $f_m=\gamma M_s/2\pi$ and $f_0=\gamma H_i/2\pi$ with γ being the gyromagnetic ratio, M_s the saturation magnetization of the ferrite, and H_i the internal dc field intensity.

The effective permeability can then be expressed as

$$\mu_{e1,2}=\frac{(f_m+f_0)^2-f_{1,2}^2}{f_0^2+f_0f_m-f_{1,2}^2}. \quad (25)$$

By using (23)–(25), it is possible to solve the equation system (20) and (21) with regard to f_m and f_0, thereby stating the requirements on the saturation magnetization and internal field intensity for circulation at the frequencies f_1 and f_2. For a surrounding material with frequency-independent properties, we get

$$f_0=\sqrt{\frac{1-\alpha_{1,2}\beta_{1,2}}{f_2^2-\alpha_{1,2}\beta_{1,2}f_1^2}\cdot\frac{(\beta_{1,2}-\alpha_{1,2})f_1^2f_2^2}{\beta_{1,2}f_2^2-\alpha_{1,2}f_1^2}} \quad (26)$$

$$f_m=\frac{\beta_{1,2}\left[f_2^4-(1+\alpha_{1,2}^2)f_1^2f_2^2+\alpha_{1,2}^2f_1^4\right]}{\sqrt{(1-\alpha_{1,2}\beta_{1,2})(f_2^2-\alpha_{1,2}\beta_{1,2}f_1^2)}\,(\beta_{1,2}f_2^2-\alpha_{1,2}f_1^2)} \quad (27)$$

and for ferrite surrounding, i.e., $Z_{d1,2}=Z_{e1,2}$,

$$f_0=\sqrt{\frac{f_2-\beta_{1,2}\alpha_{1,2}^2f_1}{f_2^3-\beta_{1,2}\alpha_{1,2}^2f_1^3}\cdot\frac{(f_2-\beta_{1,2}f_1)f_1f_2}{f_1-\beta_{1,2}f_2}} \quad (28)$$

$$f_m = \frac{\beta_{1,2}\left[f_2^4 - (1+\alpha_{1,2}^2) f_1^2 f_2^2 + \alpha_{1,2}^2 f_1^4\right]}{\sqrt{(f_2 - \beta_{1,2}\alpha_{1,2}^2 f_1)(f_2^3 - \beta_{1,2}\alpha_{1,2}^2 f_1^3)}\ (\beta_{1,2} f_2 - f_1)} \tag{29}$$

where

$$\alpha_{1,2} = \frac{X_2}{X_1} \tag{30}$$

$$\beta_{1,2} = S_{1,2} \frac{X_1 n_2^3 \sin^2(n_1\Psi) \sin(2n_1\pi/3)}{X_2 n_1^3 \sin^2(n_2\Psi)\sin(2n_2\pi/3)}. \tag{31}$$

Conversely, (20) and (21) can be solved with regard to f_1 and f_2. The relations obtained for frequency-independent surrounding material properties are

$$f_1 = \left\{ \frac{f_0 + f_m}{2\alpha_{1,2}\beta_{1,2}(\beta_{1,2}-\alpha_{1,2})} \left[\beta_{1,2}(2\alpha_{1,2}\beta_{1,2} - 1 - \alpha_{1,2}^2) f_0 + (\alpha_{1,2}\beta_{1,2}-1)(\beta_{1,2}-\alpha_{1,2}) f_m \pm A \right] \right\}^{1/2} \tag{32}$$

$$f_2 = \left\{ \frac{f_0 + f_m}{2(\beta_{1,2}-\alpha_{1,2})} \left[(\beta_{1,2} + \beta_{1,2}\alpha_{1,2}^2 - 2\alpha_{1,2}) f_0 - (\alpha_{1,2}\beta_{1,2}-1)(\beta_{1,2}-\alpha_{1,2}) f_m \pm A \right] \right\}^{1/2} \tag{33}$$

where plus or minus signs should be chosen depending on the relation between the frequencies, and

$$A = \left\{ \left[\beta_{1,2}(\alpha_{1,2}^2 - 1) f_0 - (\alpha_{1,2}\beta_{1,2}-1)(\beta_{1,2}-\alpha_{1,2}) f_m \right]^2 - 4\beta_{1,2}(\alpha_{1,2}\beta_{1,2}-1)(\beta_{1,2}-\alpha_{1,2}) f_0 f_m \right\}^{1/2}. \tag{34}$$

The expressions (32)–(34) determine the circulation frequencies obtainable for a given internal field intensity and a material with specified saturation magnetization.

The design of the double-frequency circulator is then easily completed. R can be solved from (16) or (17) giving

$$R = \frac{X_{1,2}}{2\pi f_{1,2}\sqrt{\epsilon_0 \epsilon_f \mu_0 \mu_{e1,2}}}. \tag{35}$$

Ψ is obtained from (18) or (19), whereby the influence on $\beta_{1,2}$ in (31) must be taken into account:

$$\frac{\sin^2(n_{1,2}\Psi)}{\Psi} = S_1 \frac{\pi Z_{d1,2}\kappa_{1,2} n_{1,2}^3}{2 Z_{e1,2}\mu_{1,2} X_{1,2}\sin(2n_{1,2}\pi/3)}. \tag{36}$$

A first-order approximation of the substrate thickness h in the junction is simply obtained, neglecting the stray fields at the ports. We then get

$$\frac{Z_p}{Z_{vp}} = \frac{h}{N_D w} \tag{37}$$

where Z_p and Z_{vp} are the input impedance and wave impedance, respectively, at circulation, $N_D = 1$ for microstrip and 2 for stripline, and the coupling width w is given by $w = 2R \sin\Psi$. This gives

$$h = 2N_D R \sin\Psi Z_p / Z_{vp}. \tag{38}$$

Assume that a double-frequency circulator is matched to the characteristic impedance Z_0 of the surrounding network, given by an expression similar to (37):

$$\frac{Z_0}{Z_d} = \frac{h}{N_D w}. \tag{39}$$

As $Z_{vp}(f_i) = Z_{di}$ according to (18) and (19) and the matching requires that $Z_p(f_i) = Z_0(f_i)$ for $i = 1,2$, we find from (38) and (39) that $h(f_1) = h(f_2)$. This result is, in fact, valid for TEM propagation, that is, when the right-hand sides of (37) and (39) are given by an arbitrary function $f(w/h)$.

C. Derivation of the Loaded Q Value

Defining $Z_{in}(f) = R_{in}(f) + jX_{in}(f)$, the loaded Q value is obtained by [10]

$$Q_L = \frac{f_c}{2R_{in}(f_c)} \left.\frac{dX_{in}}{df}\right|_{f=f_c} \tag{40}$$

where f_c is the resonance frequency. As shown above, see (8), (9), (14), and (15), Z_{in} in the vicinity of f_c can be written

$$Z_{in} = \frac{Z_e \sin^2(n\Psi) X |\mu|}{\pi Z_d n^3 \Psi |\kappa|} \left(\sqrt{3} - j \frac{X|\mu| J_n'(X)}{n|\kappa| J_n(X)} \right) \tag{41}$$

where in (9) it has been taken into account that $J_n'(X)^2$ is negligible compared with $[\kappa n/\mu X J_n(X)]^2$ close to resonance. By using the relation $J_n'(X) = 0$ at f_c according to (11), we get

$$\left.\frac{dX_{in}}{df}\right|_{f=f_c} = \frac{\sin^2(n\Psi)}{\pi n^4 \Psi} \left[\frac{Z_e}{Z_d}\left(\frac{\mu}{\kappa}\right)^2 (X^2 - n^2) X \left[\frac{1}{f_c} + \frac{1}{\sqrt{\mu_e}} \left.\frac{d}{df}\sqrt{\mu_e}\right|_{f=f_c} \right] \right]. \tag{42}$$

Insertion of (42) into (40) with μ_e according to (25) finally gives

$$Q_L = \frac{X^2 - n^2}{2\sqrt{3}\,n} \frac{|\mu|}{|\kappa|} \left\{ 1 + \frac{f_c^2 f_m (f_m + f_0)}{\left[(f_m+f_0)^2 - f_c^2\right]\left[f_0(f_m+f_0) - f_c^2\right]} \right\}. \tag{43}$$

By neglecting the frequency variation of μ_e, we get

$$Q_L = \frac{X^2 - n^2}{2\sqrt{3}\,n} \frac{|\mu|}{|\kappa|} \tag{44}$$

which for the (1, 1) mode coincides with earlier published results [6]. The improvement of the accuracy achieved by (43) compared with (44) is, however, often not negligible.

D. Illustration of Solutions for Double-Frequency Circulation

By using the relations (26), (27), (35), (36), and (43), we have calculated designs for double-frequency circulators. The solutions for some of the lower order mode combinations are shown in Fig. 1(a)–(f).

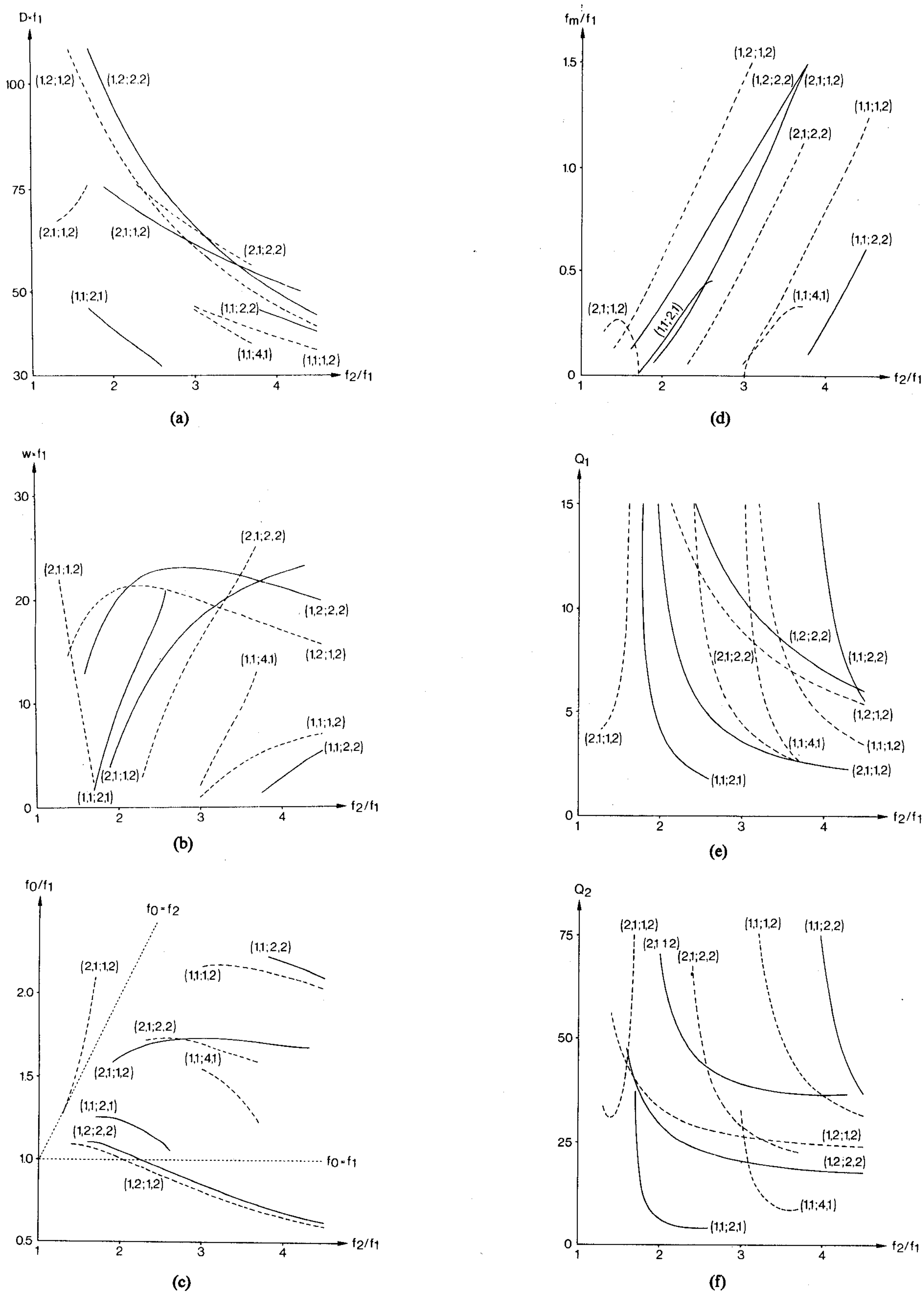

Fig. 1. Diagrams for design of double-frequency circulators using some of the lower order mode combinations. Normalized values are given for (a) diameter (D), (b) coupling width (w), (c) resonance frequency ($f_0 = \gamma H_i / 2\pi$), and (d) equivalent saturation magnetization frequency ($f_m = \gamma M_s / 2\pi$). In (e) and (f) are shown loaded Q values for the circulations. Solid lines correspond to equal circulation directions, while dashed lines indicate opposite ones. For f_1 in gigahertz, the dimensions are given in millimeters. $\epsilon_f = 14.0$; $\epsilon_d = 10.0$.

E. Conditions for Circulation at More than Two Frequencies

By generalizing the indexes introduced in Section II-B above, related to the circulations, the conditions for multiple-frequency operation can be derived analogously to (20) and (21):

$$\frac{f_{i+1}}{f_i} = \frac{X_{i+1}}{X_i}\sqrt{\frac{\mu_{ei}}{\mu_{ei+1}}} \tag{45}$$

$$\frac{X_{i+1} n_i^3 \sin^2(n_{i+1}\Psi)\mu_{i+1}\kappa_i \sin(2n_{i+1}\pi/3)}{X_i n_{i+1}^3 \sin^2(n_i\Psi)\mu_i\kappa_{i+1} \sin(2n_i\pi/3)} \cdot \sqrt{\frac{\mu_{ei+1}\mu_{di}}{\mu_{di}\mu_{di+1}}} = S_{i,i+1} \tag{46}$$

where $i=1,2,\cdots,n-1$, and n is the number of circulation frequencies. Thus for n circulation frequencies, we get a system with $2(n-1)$ equations stating the conditions to be satisfied. Comparison with the double-frequency solution shows that (45) and (46) imply the following generalized versions of (26) and (27) for frequency-independent surrounding material properties:

$$f_0 = \sqrt{\frac{1-\alpha_{i,i+1}\beta_{i,i+1}}{f_{i+1}^2 - \alpha_{i,i+1}\beta_{i,i+1}f_i^2} \cdot \frac{(\beta_{i,i+1}-\alpha_{i,i+1})f_i^2 f_{i+1}^2}{\beta_{i,i+1}f_{i+1}^2 - \alpha_{i,i+1}f_i^2}} \tag{47}$$

$$f_m = \frac{\beta_{i,i+1}\left[f_{i+1}^4 - (1+\alpha_{i,i+1}^2)f_i^2 f_{i+1}^2 + \alpha_{i,i+1}f_i^4\right]}{\sqrt{(1-\alpha_{i,i+1}\beta_{i,i+1})(f_{i+1}^2 - \alpha_{i,i+1}\beta_{i,i+1}f_i^2)}\;(\beta_{i,i+1}f_{i+1}^2 - \alpha_{i,i+1}f_i^2)} \tag{48}$$

where $i=1,2,\cdots,n-1$. $\alpha_{i,i+1}$ and $\beta_{i,i+1}$ are defined in accordance with (30) and (31):

$$\alpha_{i,i+1} = \frac{X_{i+1}}{X_i} \tag{49}$$

$$\beta_{i,i+1} = S_{i,i+1}\frac{X_i n_{i+1}^3 \sin^2(n_i\Psi)\sin(2n_i\pi/3)}{X_{i+1}n_i^3 \sin^2(n_{i+1}\Psi)\sin(2n_{i+1}\pi/3)}. \tag{50}$$

The right-hand sides of (47) and (48) state $2(n-2)$ conditions on the n circulation frequencies. By inspection of (47) and (48), it is, however, evident that a scaled solution is still a solution. Thus one frequency can always be specified, and the number of frequencies to be determined is, in fact, $n-1$. For $n=3$ we have an equal number of conditions and unknowns. The conditions for triple-frequency circulation is thus obtained by application of (47) and (48) to a double-frequency circulation at f_1 and $f_2 (i=1)$ and another one at f_2 and $f_3 (i=2)$, observing that f_0 and f_m must coincide. The solution is

$$f_2 = \left[\frac{\alpha_{1,2}(1-\alpha_{2,3}\beta_{2,3})(\alpha_{1,2}\alpha_{2,3}-\beta_{1,2}\beta_{2,3})}{(1-\alpha_{1,2}\beta_{1,2}\alpha_{2,3}\beta_{2,3})(\alpha_{2,3}-\beta_{2,3})}\right]^{1/2} f_1 \tag{51}$$

$$f_3 = \left[\frac{\alpha_{1,2}\alpha_{2,3}(1-\alpha_{2,3}\beta_{2,3})(\alpha_{1,2}-\beta_{1,2})}{(1-\alpha_{1,2}\beta_{1,2})(\alpha_{2,3}-\beta_{2,3})}\right]^{1/2} f_1. \tag{52}$$

TABLE I
TRIPLE-FREQUENCY CIRCULATORS FOR SOME OF THE LOWER ORDER MODE COMBINATIONS

			DF1	WF1	HF1	F1	F2	F3	F0	FM	Q1	Q2	Q3
11+	12-	22+	42.7	3 73	3.13	1	3 45	4 17	2.17	370	8.72	52.4	51 3
11+	12-	42-	40 5	5 14	4.31	1	3 77	6 02	2.14	.618	5 76	40.7	44.3
12+	12-	22-	101	18.3	15.4	1	1.62	1 87	1.08	.269	26.3	42.6	32 9
12+	12-	42-	98 2	19 1	16 0	1	1.69	2.53	1 07	.318	23.8	40.2	34.9
12+	12-	52+	99 2	18.8	15 8	1.	1.66	2 80	1 07	.301	24.7	41.0	40 6
21-	12+	12-	70 8	10.2	8 57	1.	1.54	2.24	1.68	.245	7.53	35.6	51.7
21-	12+	22+	71 4	9 19	7.71	1	1 56	2.58	1 73	233	8.37	37.7	42.9
21-	12-	22+	67.3	13 7	11.5	1	2 50	2.85	1.71	425	5 26	44.3	31.9

DF1, WF1, HF1 in (mm)

F1, F2, F3, F0, FM in (GHz)

(+) and (-) indicate the direction of circulation

$\varepsilon_f = 14.0$, $\varepsilon_d = 10.0$

This result establishes the fact that each combination of operating states corresponds to exactly one specified set of normalized circulation frequencies.

By using the theories given above, triple-frequency circulators have been designed for some of the lowest order mode combinations. The result is shown in Table I.

For n greater than three, the system (47) and (48) is overconditioned. Solutions are, however, possible for such mode combinations which increase the linear dependency of the equations. Circulation at n frequencies can thus be obtained by applying (51) and (52) to all combinations $f_{i-1}, f_i; i=3,4,\cdots,n$. We then get double specifications on f_i according to

$$f_i = \left[\frac{\alpha_{i-2,i-1}\alpha_{i-1,i}(1-\alpha_{i-1,i}\beta_{i-1,i})(\alpha_{i-2,i-1}-\beta_{i-2,i-1})}{(1-\alpha_{i-2,i-1}\beta_{i-2,i-1})(\alpha_{i-1,i}-\beta_{i-1,i})}\right]^{1/2} f_{i-2} \tag{53}$$

and

$$f_i = \left[\frac{\alpha_{i-1,i}(1-\alpha_{i,i+1}\beta_{i,i+1})(\alpha_{i-1,i}\alpha_{i,i+1}-\beta_{i-1,i}\beta_{i,i+1})}{(1-\alpha_{i-1,i}\beta_{i-1,i}\alpha_{i,i+1}\beta_{i,i+1})(\alpha_{i,i+1}-\beta_{i,i+1})} \cdot \frac{\alpha_{i-2,i-1}(1-\alpha_{i-1,i}\beta_{i-1,i})(\alpha_{i-2,i-1}\alpha_{i-1,i}-\beta_{i-2,i-1}\beta_{i-1,i})}{(1-\alpha_{i-2,i-1}\beta_{i-2,i-1}\alpha_{i-1,i}\beta_{i-1,i})(\alpha_{i-1,i}-\beta_{i-1,i})}\right]^{1/2} f_{i-2} \tag{54}$$

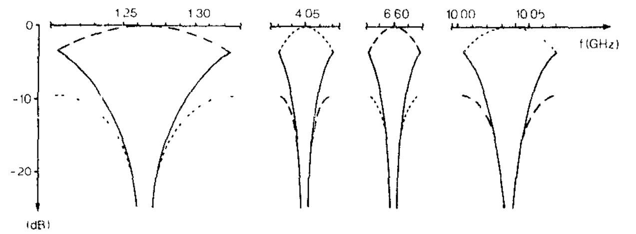

Fig. 2. Analysis of a loss-free circulator designed for operation at four frequencies. Solid, dashed, and dotted lines represent reflexion, transmission, (1-2), and (1-3).

TABLE II
COMPARISON BETWEEN DESIGNS OF A DOUBLE-FREQUENCY CIRCULATOR USING SINGLE- AND MULTIMODE DESCRIPTION.

	f_0/f_1	f_m/f_1	D (mm)	w (mm)	h (mm)
Single mode design	2.176	.228	11.94	1.38	.955
Multimode design	2.124	.236	12.31	1.46	1.01

which gives the $n-3$ conditions

$$\frac{(1-\alpha_{i-2,i-1}\beta_{i-2,i-1})(1-\alpha_{i,i+1}\beta_{i,i+1})}{(\alpha_{i-2,i-1}-\beta_{i-2,i-1})(\alpha_{i,i+1}-\beta_{i,i+1})}\cdot\frac{(\alpha_{i-2,i-1}\alpha_{i-1,i}-\beta_{i-2,i-1}\beta_{i-1,i})(\alpha_{i-1,i}\alpha_{i,i+1}-\beta_{i-1,i}\beta_{i,i+1})}{(1-\alpha_{i-2,i-1}\beta_{i-2,i-1}\alpha_{i-1,i}\beta_{i-1,i})(1-\alpha_{i-1,i}\beta_{i-1,i}\alpha_{i,i+1}\beta_{i,i+1})}=1 \tag{55}$$

for $i=3,4,\cdots,n-1$. If operation states are found such that (55) is satisfied, the circulation frequencies are obtained by (51) and by the recurrence formula (53) with $i=3,4,\cdots,n$. The design is completed by the use of the expressions given in the section for double-frequency operation above.

We have found that, for $n=4$, many of the lower order mode combinations satisfy (55) approximately. For instance, with the modes $(2,1)$ and $(1,4)$ above resonance and $(1,4)$ and $(5,5)$ below resonance the deviation between the left- and right-hand side of (55) is less than one percent. Analysis around the four circulation frequencies gave the result shown in Fig. 2.

In this section circulators with frequency-independent surrounding material have been discussed. The solution technique presented is, however, applicable also when the surrounding material has frequency-dependent properties.

III. COMPARISON WITH EXPERIMENTS

To give an indication of the validity of the approximation in taking only the dominant resonance modes into account, a circulator for double-frequency operation was built. The mode combination chosen was the $(1,1)$ mode above resonance at the lower frequency and the $(1,2)$ mode below resonance at the higher frequency. The component was first designed according to the single-mode theory. This design was then improved by using multimode description whereby the modes up to the third order were retained. The results are given in Table II.

The device was built in accordance with the multimode design. The measured performance is compared with the one given by single-mode theory in Table III.

TABLE III
COMPARISON BETWEEN CALCULATED (USING SINGLE-MODE DESCRIPTION) AND MEASURED PERFORMANCE OF THE DOUBLE-FREQUENCY CIRCULATOR

	f_1		f_2	
	meas.	calc.	meas.	calc
Insertion loss (dB)	0.4	0.5	0.4	0.5
20 dB bandwidth %	1.6	1.7	0.6	0.4
Ext. field intensity (kA/m)	240	250	240	250
Frequency quotient f_2/f_1	meas.: 3.28		calc.: 3.25	

IV. CONCLUSIONS

This paper presents a theory for design and evaluation of the properties of circulators for multiple-frequency operation. The theory is developed taking only the dominant resonance mode into account at each circulation frequency. This approximation is normally acceptable for narrow-band devices, which has been confirmed in the experimental example given. For broad-band operation, the single-mode solution can be considered as a first-order approximation of the actual one, and it can be used as a starting point for a multimode solution.

As analytic solutions to the equations describing multiple-frequency operation have been found, the computing time is negligible, a fraction of a second by the use of conventional computers. Therefore, maps of solution areas of the types exemplified in this paper which require a great number of designs are also produced inexpensively. Such maps enable rapid evaluation of possible and optimal solutions for given applications.

The loaded Q value as given by the new expression (43) has been tested against a computed one according to (13) and (40) where the corresponding difference approximation has been used to simulate the derivative. The very good correspondence obtained verifies that (41) gives an excellent approximation of the input impedance in a close vicinity of the center frequency.

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The Magnetic-Dipole Resonances of Ring Resonators of Very High Permittivity

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***Abstract*—The lowest magnetic-dipole mode with symmetry of revolution is investigated in a coaxial ring resonator of height L, inner radius b, and outer radius a. Theoretical data are given about the Q of the mode, the eigen magnetic dipole at resonance, and the structure of the fields (electric and magnetic) inside and outside the resonator. The variables are the dielectric constant $\epsilon_r = N^2$ and the dimensionless ratios b/a and $L/2a$. The data are valid in the limit of very high ϵ_r. Experiments show them to be already useful at $\epsilon_r = 35$.**

I. Introduction

THE NEED to miniaturize microwave circuits has given a new impetus to the use of dielectric resonators. Dielectric resonators are not new and were first proposed some forty years ago [1]. Their use at microwave frequencies, however, depended on the availability of materials with sufficiently high ϵ_r, sufficient temperature stability, and sufficiently low losses. Such materials now exist, and dielectric resonators find applications in microwave telecommunications systems, e.g., in 18-GHz oscillators (where they can be used instead of invar cavities), and in the design of compact waveguide and microstrip filters.

A shape which is very popular, because of its intrinsic mechanical simplicity, is the pillbox shown in Fig. 1. Most available data concern this geometry and its ϕ-independent modes. These modes belong to two families: electric-dipole modes with azimuthal $\overline{H}$ field, and magnetic-dipole modes with azimuthal $\overline{E}$ field. The former have been investigated in a previous article [2]. The latter, often called $TE_{0,1,p}$ modes, are the object of the present report.

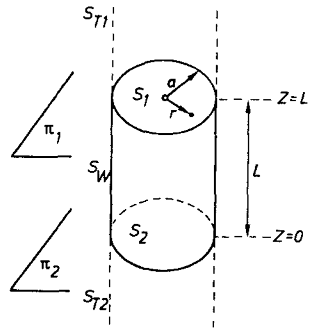

Fig. 1. Dielectric resonator (pillbox) with boundary surfaces.

The first analyses of the TE_{01p} modes [3]–[5] were conducted under the assumption that cavity walls S_1, S_2, and S_w are magnetic short circuits, i.e., that $\overline{H}$ is perpendicular to the walls. The mode components are, for such cases,

$$E_\phi = -\frac{j\omega\mu_0}{\beta} J_1(\beta r) \cos\frac{n\pi z}{L}$$

$$H_z = J_0(\beta r) \cos\frac{n\pi z}{L}$$

$$H_r = \frac{n\pi}{\beta L} J_1(\beta r) \sin\frac{n\pi z}{L} \qquad (n \text{ an integer or zero}). \quad (1)$$

Coefficient β is quantized by the boundary condition $J_0(\beta a) = 0$. The resonant frequency follows from the rela-

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